DERIBATUAK. (255. orr).

1. a) $f(x)=\dfrac{1-x}{1+x}$

$$f'(x)=\frac{-1(1+x)-(1-x)\cdot 1}{(1+x)^2}=\frac{-1-\cancel{x}-1+\cancel{x}}{(1+x)^2}=\frac{-2}{(1+x)^2}$$

b) $f(x)=\ln\dfrac{1-x}{1+x}$

$$f'(x)=\frac{1}{\frac{1-x}{1+x}}\left(\frac{1-x}{1+x}\right)'=\frac{\cancel{1+x}}{1-x}\cdot\frac{-2}{(1+x)^2}=$$

$$f'(x)=\frac{-2}{(1-x)(1+x)}=\boxed{\frac{-2}{1-x^2}}$$

LOGARITMOEN PROPIETATEEKIN

$$f(x)=\ln\frac{1-x}{1+x}=\ln(1-x)-\ln(1+x)$$

$$f'(x)=\frac{-1}{1-x}-\frac{1}{1+x}=\frac{-(1+x)-(1-x)}{1-x^2}=\boxed{\frac{-2}{1-x^2}}$$

c) $f(x)=\sqrt{\dfrac{1-x}{1+x}}$

$$f'(x)=\frac{1}{2\sqrt{\frac{1-x}{1+x}}}\cdot\left(\frac{1-x}{1+x}\right)'=\frac{1}{\cancel{2}}\sqrt{\frac{1+x}{1-x}}\;\frac{-\cancel{2}}{(1+x)^2}=\frac{-1}{\sqrt{(1-x)(1+x)^3}}$$

d) $f(x)=\dfrac{1-tg\,x}{1+tg\,x}$

$$f'(x)=\frac{-(1+tg^2x)(1+tg\,x)-(1-tg\,x)(1+tg^2x)}{(1+tg\,x)^2}$$

$$=\frac{(1+tg^2x)\left[-1-\cancel{tg\,x}-1+\cancel{tg\,x}\right]}{(1+tg\,x)^2}=\frac{-2(1+tg^2x)}{(1+tg\,x)^2}$$

②

e) $f(x) = \sqrt{\dfrac{1-tg\,x}{1+tg\,x}}$ $\quad f'(x) = \dfrac{1}{2\sqrt{\dfrac{1-tg\,x}{1+tg\,x}}} \cdot \left(\dfrac{1-tg\,x}{1+tg\,x}\right)' =$

$$= \frac{1}{\cancel{2}}\sqrt{\frac{1+tg\,x}{1-tg\,x}} \cdot \frac{-\cancel{2}(1+tg^2 x)}{(1+tg\,x)^2} = \frac{-(1+tg^2x)}{\sqrt{1-tg\,x}\,\sqrt{(1+tg\,x)^3}}$$

f) $f(x) = \ln\sqrt{e^{tg\,x}} = \dfrac{1}{2}\ln e^{tg\,x} = \dfrac{1}{2}\,tg\,x \cdot \ln e$

$$f'(x) = \frac{1}{2}\cdot \cancel{\ln e}^{\,1} \cdot (1+tg^2x) = \frac{1+tg^2x}{2}$$

g) $f(x) = \sqrt{3^{x+1}}$ $\quad f'(x) = \dfrac{1}{2\sqrt{3^{x+1}}}\,(3^{x+1})' =$

$$f'(x) = \frac{1}{2\sqrt{3^{x+1}}}\cdot 3^{x+1}\cdot \ln 3 \cdot 1 = \frac{\ln 3}{2}\cdot\sqrt{3^{x+1}}$$

h) $f(x) = \log(\sin x \cdot \cos x)^2 = 2(\log \sin x + \log \cos x)$

$$f'(x) = 2\cdot\left(\frac{1}{\sin x \cdot \ln 10}\cdot \cos x + \frac{1}{\cos x \ln 10}\cdot(-\sin x)\right) =$$

$$= \frac{2}{\ln 10}\cdot\frac{\cos^2 x - \sin^2 x}{\sin x \cdot \cos x}$$

i) $f(x) = tg^2 x + \sin^2 x$

$$f'(x) = 2\,tg\,x \cdot (1+tg^2x) + 2\sin x \cdot \cos x$$

j.) $f(x) = \underbrace{\sin\sqrt{x+1}}\cdot\underbrace{\cos\sqrt{x-1}}$

$f'(x) = \cos\sqrt{x+1}\cdot\left(\sqrt{x+1}\right)'\cdot\cos\sqrt{x-1} - \sin\sqrt{x+1}\cdot\sin\sqrt{x-1}\left(\sqrt{x-1}\right)'$

$$= \frac{\cos\sqrt{x+1}\cdot\cos\sqrt{x-1}}{2\sqrt{x+1}} + \frac{\sin\sqrt{x+1}\cdot\sin\sqrt{x-1}}{2\sqrt{x-1}}$$

k) $f(x) = \arcsin\sqrt{x}$

$$f'(x) = \frac{1}{\sqrt{1-(\sqrt{x})^2}}\cdot(\sqrt{x})' = \frac{1}{\sqrt{1-x}}\,\frac{1}{2\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x-x^2}}}$$

e.) $f(x) = \sin\left(3x^5 - 2\sqrt{x} + \sqrt[3]{2x}\right) =$ $\qquad \sqrt[3]{x} \to \frac{1}{3}x^{\frac{1}{3}-1}$

$= \sin\left(3x^5 - 2\sqrt{x} + \sqrt[3]{2}\cdot\sqrt[3]{x}\right)$

$$f'(x) = \cos\left(3x^5 - 2\sqrt{x} + \sqrt[3]{2x}\right)\cdot\left(15x^4 - \frac{2}{2\sqrt{x}} + \frac{\sqrt[3]{2}}{3\sqrt[3]{x^2}}\right)$$

m.) $f(x) = \sqrt{\sin x + x^2 + 1}$

$$f'(x) = \frac{1}{2\sqrt{\sin x + x^2 + 1}}\cdot(\cos x + 2x)$$

n.) $f(x) = \cos^2\sqrt[3]{x+(3-x)^2} = \left[\cos\sqrt[3]{x+(3-x)^2}\right]^2$

$$f'(x) = \underbrace{2\cdot\cos\sqrt[3]{x+(3-x)^2}\cdot\sin\sqrt[3]{x+(3-x)^2}}\cdot\frac{1}{3\sqrt[3]{(x+(3-x))^2}}\cdot\left(1-2(3-x)\right)$$

$$= \frac{(5-2x)\sin\left(2\cdot\sqrt[3]{x+(3-x)^2}\right)}{2\sqrt[3]{\left(x+(3-x)^2\right)^2}}$$